Florida Department of Education
Adult General Education
Curriculum Framework

ADULT BASIC EDUCATION-MATHEMATICS	
Program Title	Adult Basic Education (ABE)
Program Number	9900000
Course Title	Adult Basic Education-Mathematics
Course Number	School Districts: 9900001 Florida College System: ABX0100-ABX0199
CIP Number	1532010100
Grade Equivalent	0.0 – 8.9
Grade Level	30, 31
Standard Length	Varies (See Program Lengths Section)

PURPOSE
The Adult Basic Education (ABE) Program includes content standards that describe what students should know and be able to do in Mathematics, Language Arts (language, speaking and listening, and writing), and Reading. The content standards serve several purposes:

- Provide a common language for ABE levels among programs
- Assist programs with ABE curriculum development
- Provide guidance for new ABE instructors
- Ensure quality instruction through professional development
- Provide basic skills instruction (0.0 – 8.9) and critical thinking skills to prepare students for GED preparation (9.0 – 12.9), postsecondary education, and employment.

The content standards should be used as a basis for curriculum design and also to assist programs and teachers with selecting or designing appropriate instructional materials, instructional techniques, and ongoing assessment strategies.

The ABE content standards have been revised to include the State standards. The integration of CCR standards into ABE programs is intended to provide the foundation of knowledge and skills that students will need to transition to adult secondary programs with the goal of continuing on to postsecondary education.

PRPGRAM STRUCTURE
ABE is a non-credit course designed to develop literacy skills necessary for students to be successful workers, citizens, and family members. A student enrolled in the ABE program may be receiving instruction in one or more of the following courses: Mathematics, Language Arts, or Reading.

This program is divided into levels that are reported as student educational gains: Educational Functioning Levels (EFLs) for federal reporting and Literacy Completion Points (LCPs) for state reporting.

Progress through levels must be measured by approved validation methods in accordance with Rule 6A-6.014, F.A.C.

PROGRAM LENGTHS

The following table illustrates the recommended maximum number of instructional hours for each level. It is understood; however, that each student learns at his or her individual pace, and there will be students who successfully complete the program or attain their educational goals in fewer or more hours than what is recommended for each ABE instructional level.

Please visit the Assessment Technical Assessment Paper, Division of Career and Adult Education, at http://www.fldoe.org/academics/career-adult-edu/adult-edu/technical-assistance-papers.stml for both recommended and required assessment procedures and instruments.

Course Number	Course Title	Maximum Hours	NRS Levels
9900001 ABX0100-ABX0199	Mathematics – ABE Level One (1)	450 Hours	**1** (0.0 – 1.9)
	Mathematics – ABE Level Two (2)	450 Hours	**2** (2.0 – 3.9)
	Mathematics – ABE Level Three (3)	300 Hours	**3** (4.0 – 5.9)
	Mathematics – ABE Level Four (4)	300 Hours	**4** (6.0 – 8.9)

SPECIAL NOTES

The mathematic standards are separated into ten strands as shown in the chart below. Each strand is headed by a strand-specific set of CCR anchor standards identical across all levels of learning. Each level-specific standard corresponds to the same-numbered CCR anchor standard. In other words, each anchor standard identifying broad state skills has a corresponding level-specific standard illustrating specific level-appropriate expectations call a benchmark skill. The table below illustrates the numbering used to indicate strands, anchor standards, and skill standards.

Source	Strand	Program Area	Mathematic Domain	NRS Level	Anchor Standard	Benchmark Skill
	MA.	ABE.	2.	1.	3.	a)
MA.ABE.2. **Operations and Algebraic Thinking** **1.3 Add and subtract with 20.** a) Relate counting to addition and subtraction by counting by 2 to add or subtract by 2.						

It is not intended that students will progress through the performance standards sequentially. The instructor may present topic-centered and/or project-based lessons that integrate standards from several academic strands.

ADULT EDUCATION INSTRUCTOR CERTIFICATION

As per section 1012.39 (1)(b), F.S., each school district shall establish the minimal qualifications for part-time and full-time teachers in adult education programs.

ACCOMODATIONS

Federal and state legislation requires the provision of accommodations for students with disabilities to meet individual needs and ensure equal access. **Adult students with disabilities must self-identify, provide documentation, and request such services.** Students with disabilities may need accommodations in areas such as instructional methods and materials, assignments and assessments, time demands and schedules, learning environment, assistive technology, and special communication systems. Documentation of the accommodations requested and provided should be maintained in a confidential file.

CAREER AND EDUCATION PLANNING

The following career development standards are designed to be integrated into the ABE frameworks to assist students with career exploration and planning. Students can access the local agency's approved career information program for career exploration and development of a career plan.

Standards

CP. ABE.01	Develop skills to locate, evaluate, and interpret career information.
CP. ABE.02	Identify interests, skills, and personal preferences that influence career and education choices.
CP. ABE.03	Identify career cluster and related pathways that match career and education goals.
CP. ABE.04	Develop and manage a career and education plan.

DIGITAL LITERACY (TECHNOLOGY)

Computer skills have become essential in today's world. Students use a variety of technology tools such as calculators, cell phones, and computers for multiple uses; communicate with friends and family, apply for work, classroom instruction, testing, and in the workplace. Technology standards are integrated in the instruction to demonstrate proficiency of the reading and language arts standards. (Example standards: Mathematics 4, Reading 7, Writing 6, and Speaking and Listening 5).

Standards

DL. ABE.01	Develop basic keyboarding and numerical keypad skills.
DL. ABE.02	Produce a variety of documents such as research papers, resumes, charts, and tables using word processing programs.
DL. ABE.03	Use Internet search engines such as Google, Bing, or Yahoo to collect data and information.
DL. ABE.04	Practice safe, legal, and responsible sharing of information, data, and opinions online.

WORKFORCE PREPARATION ACTIVITIES

The term "workforce preparation activities" means activities, programs, or services designed to help an individual acquire a combination of basic academic skills, critical thinking skills, digital literacy skills, and self-management skills, including competencies in utilizing resources, using information, working with others, understanding systems, and obtaining skills necessary for successful transition into and completion of postsecondary education or training, or employment. (Workforce Innovation and Opportunity Act (WIOA), 2014).

The following activities should be integrated into the classroom instruction:

Critical Thinking	All students will make decisions and solve problems by specifying goals, identifying resources and constraints, generating alternatives, considering impacts, choosing appropriate alternatives, implementing plans of action, and evaluating results.
Teamwork	All students will learn to work cooperatively with people with diverse backgrounds and abilities. Students will identify with the group's goals and values, learn to exercise leadership, teach others new skills, serve clients or customers, and contribute with ideas, suggestions, and work efforts.
Employment	All students will develop job search skills for employment such as completing an application, resume, cover letter, thank you letter, and interviewing techniques.
Self-Management	All students should display personal qualities such as responsibility, self-management, self-confidence, ethical behavior, and respect for self and others.
Utilizing Resources	All students will learn to identify, organize, plan, and allocate resources (such as time, money, material, and human resources) efficiently and effectively.
Using Information	All students will acquire, organize, interpret, and evaluate information in post-secondary, training, or work situations.
Understanding Systems	All students will learn to understand, monitor, and improve complex systems, including social, technical, and mechanical systems, and work with and maintain a variety of technologies.

INTEGRATED EDUCATION AND TRAINING (IET)

DCAE promotes the planning, development and implementation of an integrated education and training (IET) service approach that provides concurrent and contextualized adult education and literacy activities in combination with workforce preparation activities and workforce training for a specific occupation or occupational cluster for the purpose of educational and career advancement.

The IET service approach provides all levels of adult education students the opportunity to acquire the skills needed to:

- Transition to and complete postsecondary education and training programs;
- Obtain and advance in employment leading to economic self-sufficiency; and
- Exercise the rights and responsibilities of citizenship.

All IET programs must include the following three components:

- Adult education and literacy activities (§463.30);
- Workforce preparation activities (§463.34); and
- Workforce training for a specific occupation or occupation cluster which can be any one of the training services defined in section 134(c)(3)(D), of WIOA.

In order to meet the "integrated" requirement of IET, all services must include the following:

- Adult education and literacy activities run concurrently and contextually with workforce preparation activities and workforce training for a specific occupation or occupational cluster for the purpose of educational and career advancement;
- Activities are of sufficient intensity and quality, and based on the most rigorous research available, particularly with respect to improving reading, writing, mathematics, and English proficiency of eligible individuals;
- Occur simultaneously; and
- Use occupational relevant instructional materials.

The integrated education and training program must have a single set of learning objectives that identifies specific adult education content, workforce preparation activities, and workforce training competencies, and the program activities function cooperatively.

ABE MATHEMATICAL STANDARDS

The chart below provides an overview of the ten domains that comprise Florida's ABE mathematic standards across instruction levels. The mathematic standards are presented into two broad instructional groupings; 1) basic literacy and, 2) intermediate. Basic literacy includes NRS levels 1 and 2 (grade equivalent (GE: 0.0 – 3.9) and intermediate includes NRS levels 3 and 4 (GE: 4.0 – 8.9).

Each instructional level has a limited number of anchor standards. This allows mathematical instruction at each NRS level to have a narrow and deep focus that allows the student to develop an understanding of mathematical foundations, conceptual understandings, procedural skills, and fluency. The chart's shaded areas indicate that the domain does not have an anchor standard or primary focus for instruction at that particular instructional level. While the anchor standards by design guide instruction, teachers may introduce, practice, reinforce, and develop fluency at lower and/or higher instructional levels. Two domains, fractions and functions, have been noted (*) because the suggested instruction should begin at the mid-point of the NRS level.

ADULT BASIC EDUCATION MATHEMATIC DOMAINS					
Domain Number	**NRS Reporting Grade Equivalent (GE)**	**NRS Level 1 0.0 – 1.9**	**NRS Level 2 2.0 – 3.9**	**NRS Level 3 4.0 – 5.9**	**NRS Level 4 6.0 – 8.9**
1	Number and Operations: Base Ten	0.0 – 1.9	2.0 – 3.9	4.0 – 5.9	
2	Operations and Algebraic Thinking	0.0 – 1.9	2.0 – 3.9	4.0 – 5.9	
3	Measurement and Data	0.0 – 1.9	2.0 – 3.9	4.0 – 5.9	
4	Geometry	0.0 – 1.9	2.0 – 3.9	4.0 – 5.9	6.0 – 8.9
5	Number and Operations: Fractions		*3.0 – 3.9	4.0 – 5.9	
6	Expressions and Equations			4.0 – 5.9	6.0 – 8.9

7	The Number System			4.0 – 5.9	6.0 – 8.9
8	Ratios and Proportional Relationships			4.0 – 5.9	6.0 – 8.9
9	Statistics and Probability			4.0 – 5.9	6.0 – 8.9
10	Functions				*7.0 – 8.9

MATHEMATICS (MA) Basic Literacy, GE: 0.0 – 3.9

Mathematics Standards NRS Level 1
Beginning ABE Literacy, GE 0.0 – 1.9

Mathematics instruction begins with basic literacy skills. The primary focus of level 1 is counting, cardinality, number sense, and base-ten operations. Students at this level are developing their understanding of whole number relationships, linear measurement (length), two-digit place value, and strategies for addition and subtraction.

This level begins building a basic foundation for algebra by introducing the concept of an equation, a variable, and the meaning of the equal sign, all within the context of addition and subtraction within 20.

Lastly, instruction provides some attention to describing and reasoning geometric shapes as a basis for understanding the properties of congruence, similarity, and symmetry.

Mathematics Standards NRS Level 2
Beginning Basic Education, GE: 2.0 – 3.9

NRS level 2 emphasizes understanding place value for whole numbers to 1000, developing fluency in addition and subtraction to 3 digits, understanding and exploring strategies for multiplication and division within 100, and a crucial foundation for fractions. These skills prepare students for work with rational numbers, ratios, rates, and proportions in subsequent levels.

In the areas of measurement and geometry, using standard units of measure and developing understanding of the structure of rectangular arrays and areas are priorities, as well as analyzing two-dimensional shapes as a foundation for area, volume, congruence, similarity and symmetry.

MATHEMATICS (MA) Basic Literacy GE: 0.0-3.9 Anchor Standards and Benchmark Skills	
NRS LEVEL 1 GE: 0.0 – 1.9	**NRS LEVEL 2 GE: 2.0 – 3.9**

MA.ABE.1. **Number and Operations: Base Ten**	
1.1 Understand place value of two-digit numbers. a) Understand that the two digits of a two-digit number represent amounts of tens and ones. b) Compare two two-digit numbers recording the results of comparisons with the symbols greater than (>), equal to (=), and less than (<).	**2.1 Understand place value of three-digit numbers.** a) Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. b) Count within 1000 by 5s, 10s, and 100s. c) Read and write numbers to 1000 using numerals, number names, and expanded form. d) Compare two three-digit numbers using greater than (>), equal to (=), and less than (<) symbols to record the results of comparisons.
1.2 Use place value understanding and the properties of operations to add and subtract within 100. a) Add within 100, including adding a two digit number and a one-digit number, two-digit numbers, and multiples of 10. b) Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose (create) a ten. c) Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count. d) Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences). e) Use concrete models, drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used.	**2.2 Use place value understanding and properties of operations to add and subtract within 1000.** a) Add within 1000 up to four two-digit numbers using strategies based on place value and properties of operations. b) Understand that in adding or subtracting three-digit numbers, sometimes it is necessary to compose (put together) or decompose (take apart) tens or hundreds. c) Mentally add or subtract 10 or 100 to a given number 100–900. d) Use concrete models, drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. e) Explain why addition and subtraction strategies work, such as using place value and the properties of operations.
	2.3 Use place value understanding and properties of operations to perform multi-digit arithmetic. a) Use place value to round whole numbers to the nearest 10 or 100. b) Fluently add and subtract within 1000 using strategies and algorithms (step-by-step procedure for calculation) based on place value, properties of operations, and/or the relationship between addition and subtraction. c) Multiply one-digit whole numbers by multiples of 10 in the range 10–90, using strategies based on place value and properties of operations.
NRS LEVEL 1 **GE: 0.0 – 1.9**	**NRS LEVEL 2** **GE: 2.0 – 3.9**
MA.ABE.2. **Operations and Algebraic Thinking**	

1.1 Represent and solve problems involving addition and subtraction within 20. a) Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 by using objects, drawings, and equations (statement that says two expressions are equal) with a symbol for the unknown number to represent the problem.	**2.1 Represent and solve problems involving addition and subtraction within 100.** a) Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions by using drawings and equations with a symbol for the unknown number to represent the problem.
1.2 Understand and apply properties of operations and the relationship between addition and subtraction. a) Apply properties of operations as strategies to add and subtract. • Commutative property of addition. • Associative property of addition. b) Understand subtraction as an unknown-addend problem.	**2.2 Fluently add and subtract within 20.** a) Fluently add and subtract within 20 using mental strategies. b) Know from memory sums of 2 one-digit numbers (math facts 0-9).
1.3 Add and subtract with 20. b) Relate counting to addition and subtraction by counting by 2 to add or subtract by 2. c) Add and subtract within 20 using strategies such as: • Counting on. • Making ten. • Decomposing (taking apart) a number leading to a ten. • Using the relationship between addition and subtraction. • Creating equivalent but easier known sums.	**2.3 Represent and solve problems involving multiplication and division.** a) Interpret products of numbers, such as 5x7 as the total number of objects in 5 groups of 7 objects each. b) Interpret quotients of numbers, such as, 56÷8 as the number of objects in a share. c) Use multiplication and division within 100 to solve word problems using drawings and equations with a symbol for the unknown number to represent the problem. d) Determine the unknown number in a multiplication or division equation relating three numbers.
1.4 Work with addition and subtraction equations. a) Understand the meaning of the equal sign and determine if equations are true or false. b) Determine the unknown number in an equation relating three whole numbers.	**2.4 Understand properties of multiplication and the relationship between multiplication and division.** a) Apply properties of operations as strategies to multiply and/or divide: • Commutative property of multiplication. • Associative property of multiplication. • Distributive property of multiplication. b) Understand division as an unknown-factor problem.
	2.5 Multiply and divide within 100. a) Fluently multiply and divide within 100. b) Use strategies such as the relationship between multiplication and division or properties of operations. c) Know from memory products of two one-digit numbers (math facts 0-9).
	2.6 Solve problems involving the four operations, and identify and explain patterns in arithmetic.

	a) Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. b) Identify arithmetic patterns, including patterns in the addition table or multiplication table, and explain them using properties of operations.
NRS LEVEL 1 **GE: 0.0 – 1.9**	**NRS LEVEL 2** **GE: 2.0 – 3.9**
MA.ABE.3. **Measurement and Data**	
1.1 Represent and interpret data. a) Organize, represent, and interpret data with up to three categories. • Ask and answer questions about the total number of data points. • How many are represented in each category. • How many more or less are represented in one category than in another.	**2.1 Represent and interpret data.** a) Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. • Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. b) Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. • Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. c) Create a line plot to represent data. • Generate measurement data by using measuring tools marked with halves and fourths of a unit of measure (ruler). • Show the data by making a line plot, where the horizontal scale is marked off in units (whole numbers, halves, or fourths).
1.2 Measure lengths indirectly and by iterating (repeating) length units. a) Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end. b) Understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	**2.2 Measure and estimate lengths in standard units.** a) Compare and describe how using standard (ruler) and nonstandard (thumb) units of measure relate to the size of the unit chosen. b) Estimate lengths using units of inches, feet, centimeters, and meters. c) Measure to determine how much longer one object is than another, using a standard length unit.
	2.3 Relate addition and subtraction to length. a) Represent whole numbers as lengths from 0 on a number line diagram. b) Represent whole number sums and differences within 100 on a number line diagram.
	2.4 Solve problems involving measurement and

	estimation of intervals of time, liquid volumes, and masses of objects. a) Measure, tell, and write time to the nearest minute. b) Solve word problems involving addition and subtraction of time intervals in minutes by representing the problem on a number line diagram. c) Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). d) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, by using drawings, such as a beaker with a measurement scale, to represent the problem.
	2.5 Understand concepts of area measurement and relate area to multiplication and addition. a) Recognize area as an attribute of plane figures and understand concepts of area measurement. • A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. • A plane figure which can be covered without gaps or overlaps by (*n)* unit squares is said to have an area of (*n)* square units. b) Measure areas by counting unit squares (square cm., square m., square in., square ft., and non-specific units). c) Relate area to the operations of multiplication and addition. • Use math tiles to find the area of a rectangle and show that the area is the same as by multiplying the side lengths. • Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems. • Use area models to represent the distributive property in mathematical reasoning. • Use math tiles to show that the area of a rectangle with whole number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. d) Recognize area as additive. Find areas of rectilinear figures (formed by straight lines) by decomposing them into non-overlapping rectangles and adding the areas.
	2.6 Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

	a) Solve real world and mathematical problems involving perimeters of polygons. • Find the perimeter given the side lengths. • Find an unknown side length. • Exhibit rectangles with the same perimeter and different areas or with the same area and different perimeters.
NRS LEVEL 1 **GE: 0.0 – 1.9**	**NRS LEVEL 2** **GE: 2.0 – 3.9**
MA.ABE.4. **Geometry**	
1.1 Analyze, compare, and create (compose) shapes. a) Analyze and compare two- and three-dimensional shapes that are different sizes and orientations. b) Use informal language to describe: • Their similarities and differences. • Their parts such as the number of sides and vertices/corners. • Other attributes such as having sides of equal length.	**2.1 Analyze and compare angles within shapes.** a) Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. b) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
1.2 Reason with composite shapes and their attributes. a) Compose (create) two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape (the shape of a house is made from a square and triangle), and new shapes from the composite shape.	**2.2 Reason with shapes and their attributes.** a) Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., b) Recognize that equal shares of identical wholes need not have the same shape. b) Understand that shapes in different categories (rhombuses, rectangles, and others) may share attributes (having four sides), and that the shared attributes can define a larger category (quadrilaterals). • Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. c) Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.
NRS LEVEL 1 **GE: 0.0 – 1.9**	**NRS LEVEL 2** **GE: 2.0 – 3.9**
MA.ABE.5. **Number and Operations: Fractions** **Note: Suggested instructional level begins at 3.0**	
Not a focus standard at this level.	**2.1 Develop understanding of fractions as numbers using denominators of 2, 3, 4, 6, or 8.** a) Understand a fraction as the quantity formed when a

	whole is partitioned into equal parts. b) Understand a fraction as a number on the number line; represent fractions on a number line diagram. • Represent a fraction on a number line diagram by defining the interval from 0 to 1 (endpoints) and partitioning it into equal parts. • Explain a fraction on a number line diagram has the interval size *a/b.*
	2.2 Develop understanding of equivalent fractions. a) Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. • Conclude two fractions as equivalent (equal) if they are the same size, or the same point on a number line. • Generate simple equivalent fractions, (1/2 = 2/4, 4/6 = 2/3) by using a visual fraction model. • Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. b) Compare two fractions with the same numerator or the same denominator. • Recognize that comparisons are valid only when the two fractions refer to the same whole. • Record the results of comparisons with the symbols >, =, or <, and justify the conclusions with a visual fraction model.

MATHEMATICS (MA) Intermediate, GE: 4.0 – 8.9

Mathematics Standards NRS Level 3
Low Intermediate Basic Education, GE: 4.0 – 5.9

NRS level 3 provides the mathematical fundamentals for all higher mathematical studies. The focus standards for this instructional level provide a conceptual foundation for learning functions. The emphasis continues on standards for numbers and operations, however, attention to algebra and geometry increase considerably.

Fluency with multi-digit whole and decimal numbers as well as calculations with fractions and the relationships between them is critical at this level. This extends to working with the concept of ratio and rates, addition and subtraction of fractions, and understanding why the procedures for multiplying and dividing fractions make sense.

Students at level 3 generate patterns in numbers and shapes in addition to reading, writing, and interpreting expressions and equations. In addition, analyzing geometric properties, such as parallelism, perpendicularity, and symmetry, and developing and finding volumes of right rectangular prisms take precedence.

Measurement and data instruction shifts to sampling techniques and data collection through statistical questioning; to previous standards about data, it adds the understanding of measures of center and spread and display of collected data with line plots.

Mathematics Standards NRS Level 4
High Intermediate Basic Education, GE: 6.0 – 8.9

Like preceding levels, NRS level 4 also emphasizes number sense and operations, but here the attention is on fluency with all four operations with rational numbers—both negative and positive. The foundation for understanding of irrational numbers is built here, including calculation with square and cube roots and solving simple quadratic equations.

Another area of concentration is algebra and functions: formulating and reasoning about expressions, equations, and inequalities; solving linear equations and systems of linear equations; grasping the concept of a function; and using functions to describe quantitative relationships.

Building on the geometric analysis in level 3, the focus turns to analyzing two- and three-dimensional figures using distance, angle, similarity, and congruence, and understanding basic right triangle trigonometry.

NRS level 4 is where understanding and applying ratios, rates, and proportional reasoning are developed and a bridge between rational number operations and algebraic relationships is created.

Having worked with measurement data in previous levels, students at this level develop notions of statistical variability and learn to understand summary statistics and distributions.
The concept of probability is introduced and developed at this level.

<table>
<tr><th colspan="2">MATHEMATICS (MA) Intermediate
GE: 4.0 – 8.9
Anchor Standards and Benchmark Skills</th></tr>
<tr><th>NRS Level 3
GE: 4.0 – 5.9</th><th>NRS Level 4
GE: 6.0 – 8.9</th></tr>
<tr><td colspan="2">MA.ABE.1.
Number and Operations: Base Ten</td></tr>
<tr><td>3.1 Generalize place value understanding for multi-digit whole numbers.
a) Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
b) Read and write multi-digit whole numbers using numerals, names, and expanded form.
c) Compare two multi-digit numbers based on the digits in each place, using greater than (>), equal to (=), and less than (<) symbols.
d) Use place value to round multi-digit whole numbers</td><td>Not a focus standard at this level.</td></tr>
</table>

to any place.	
3.2 Use strategies based on place value understanding and properties of operations to perform multi-digit arithmetic. a) Fluently add and subtract multi-digit whole numbers using the standard algorithm (step-by-step procedure). b) Multiply a whole number of up to four digits by one-digit and two two-digit numbers. • Illustrate and explain the calculation by using equations (statement that says two expressions are equal), rectangular arrays (displays), and/or area models. c) Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. • Use the relationship between multiplication and division. • Illustrate and explain the calculation by using equations, and/or geometry.	
3.3 Use the place value system to understand decimals. a) Recognize that a digit represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. b) Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. c) Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. • Use whole-number exponents to denote powers of 10. d) Read, write, and compare decimals to thousandths. • Read and write decimals to thousandths using numerals, names, and expanded form. • Compare two decimals to thousandths based on the digits in each place, using >, =, and < symbols to record the results of comparisons. e) Use place value understanding to round decimals to any place.	
3.4 Perform operations with multi-digit whole numbers and with decimals to hundredths. a) Fluently multiply multi-digit whole numbers using the standard algorithm. b) Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors by using strategies based on place value, the properties of operations, and/or the relationship	

between multiplication and division. • Illustrate and explain the calculation by using equations, geometry, and/or models. c) Add, subtract, multiply, and divide decimals to hundredths by using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • Relate the strategy to a written method and explain the reasoning used. • Use financial literacy applications.	
NRS Level 3 **GE: 4.0 – 5.9**	**NRS Level 4** **GE: 6.0 – 8.9**
MA. ABE 2. **Operations and Algebraic Thinking**	
3.1 Use the four operations with whole numbers to solve problems. a) Interpret a multiplication equation as a comparison statement, interpret 35 = 5×7 as 35 is 5 times as many as 7 and 7 times as many as 5. • Represent verbal statements of multiplicative comparisons as multiplication equations. b) Multiply or divide to solve word problems involving multiplicative comparison by using drawings and equations with a symbol for the unknown number to represent the problem to distinguishing multiplicative comparison from additive comparison. c) Solve multi-step word problems using the four operations, including problems in which remainders must be interpreted. • Use equations with a letter standing for the unknown quantity. • Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**Not a focus standard at this level.** **Refer to expressions and equations (page 21) and functions (page 29).**
3.2 Determine factors and multiples. a) Find all factor pairs for a whole number in the range 1–100. b) Recognize that a whole number is a multiple of each of its factors. c) Determine whether a whole number in the range 1–100 is a multiple of a one-digit number. d) Determine whether a whole number in the range 1–100 is prime or composite.	
3.3 Generate and analyze patterns. a) Generate a number or shape pattern that follows a given rule.	

b) Identify apparent features of the pattern that were not explicit in the rule itself.	
3.4 Write and interpret numerical expressions. a) Use parentheses, brackets, or braces in expressions, and evaluate expressions with these symbols. b) Write simple expressions that record calculations with numbers, and interpret expressions without evaluating them.	
NRS Level 3 **GE: 4.0 – 5.9**	**NRS Level 4** **GE: 6.0 – 8.9**
MA.ABE.3. **Measurement and Data**	
3.1 Solve problems involving measurement and conversion from a larger unit to a smaller unit. a) Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money. • Include problems involving simple fractions or decimals. • Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. b) Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	**Not a focus standard at this level.** **Refer to statistics and probability (page 26).**
3.2 Convert like measurement units within a given measurement system. a) Convert among different-sized standard measurement units (km., m., cm., kg., g., lb., oz., l., ml., hr., min., sec.), within a measurement system, such as convert 5 cm to 0.05 m, and use these conversions in solving multi-step, real world problems.	
3.3 Represent and interpret data. a) Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). b) Solve problems involving information presented in line plots. c) Use plots of numbers other than measurements.	
3.4 Demonstrate concepts of angles and measure angles. a) Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. b) Understand concepts of angle measurement: • An angle is measured to a circle with its center the common endpoint of the rays and the fraction of the circular arc between the points	

where the two rays intersect the circle. • An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. • An angle that turns through (*n)* one-degree angles is said to have an angle measure of (*n)* degrees. c) Measure and sketch angles in whole-number degrees using a protractor. d) Recognize angle measure as additive. When an angle is decomposed (broken) into non-overlapping parts, the angle measure is the sum of the parts. e) Solve addition and subtraction problems to find unknown angles on a diagram by using an equation with a symbol for the unknown angle measure.	
3.5 Apply concepts of volume measurement and relate volume to multiplication and to addition of whole numbers. a) Recognize volume as an attribute of solid figures and understand concepts of volume measurement. • A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. • A solid figure which can be packed using (*n)* unit cubes is said to have a volume of (*n)* cubic units. b) Measure volumes by counting unit cubes, using cubic cm., cubic in., cubic ft., and improvised units. c) Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. • Find the volume of a right rectangular prism by packing it with unit cubes, show that the same volume would be found by multiplying the edge lengths and by multiplying the height by the area of the base. • Represent threefold products as volumes such as the associative property of multiplication. • Apply the formulas $V = L \times W \times H$ and $V = B \times H$ for rectangular prisms to find volumes of right rectangular prisms edge lengths. • Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes; apply this technique to solve real world problems.	
NRS Level 3 **GE: 4.0 – 5.9**	**NRS Level 4** **GE: 6.0 – 8.9**

<table>
<tr><th colspan="2">MA.ABE.4.
Geometry</th></tr>
<tr><td>3.1 Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
a) Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</td><td>4.1 Draw, construct, and describe geometrical figures and describe the relationships between them.
a) Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</td></tr>
<tr><td>3.2 Graph points on the coordinate plane to solve mathematical and real-world problems.
a) Use a pair of perpendicular number lines, (axis/axes), with the intersection of the lines (the origin) arranged at 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.
• Demonstrate the first number indicates how far to move from the origin in the direction of one axis.
• Demonstrate the second number indicates how far to move in the direction of the second axis.
• Name and/or label the two axes and the coordinates correspond (x-axis and x-coordinate, y-axis and y-coordinate).
b) Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</td><td>4.2 Solve mathematical and real-world problems involving angle, measure, area, surface area, and volume.
a) Know the formulas for the area and circumference of a circle and use them to solve problems.
• Give an informal derivation (example) of the relationship between the circumference and area of a circle.
b) Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
c) Solve problems involving area, volume and surface area of two- dimensional and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</td></tr>
<tr><td>3.3 Classify two-dimensional figures into categories based on their properties.
a) Observe that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.</td><td>4.3 Produce congruence and similarity using physical models, transparencies, or geometry software.
a) Show that a two-dimensional figure is congruent (same shape and size) to another if the shapes can be obtained by a sequence of rotations (circular movement), reflections (mirror image), translations (slide).
b) Given two congruent figures, describe a sequence that exhibits the congruence between them.
c) Show that a two-dimensional figure is similar to another if the shapes can be obtained by a sequence of rotations, reflections, and translations and dilations (resize).
d) Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
e) Discuss and establish facts about:
• The angle sum and exterior angle of triangles.
• The angles created when parallel lines are cut by a transversal (a line that crosses lines).</td></tr>
</table>

	• The angle-angle criterion for similarity of triangles.
3.4 Solve mathematical and real-world problems involving area, surface area, and volume. a) Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. b) Draw polygons in the coordinate plane given coordinates for the vertices. • Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. c) Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures.	**4.4 Explain and apply the Pythagorean Theorem.** a) Apply the Pythagorean Theorem ($a^2 + b^2 = c^2$) to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. b) Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
NRS Level 3 **GE: 4.0 – 5.9**	**NRS Level 4** **GE: 6.0 – 8.9**
MA.ABE.5. **Number and Operations: Fractions**	
3.1 Extend understanding of fraction equivalence and ordering. a) Explain why a fraction *a*/*b* is equivalent to a fraction (*n* x *a*)/(*n* x *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. • Use this principle to recognize and generate equivalent fractions. b) Compare two fractions with different numerators and denominators by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. • Recognize that comparisons are valid only when the two fractions refer to the same whole. • Record the results of comparisons with symbols >, =, or <, and justify the conclusions, such as using a visual fraction model.	**Not a focus standard at this level.**
3.2 Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers. a) Construct a fraction *a*/*b* with *a*>1 as a sum of fractions 1/*b*. • Add and subtract fractions by joining and separating parts referring to the same whole. • Decompose (take apart) a fraction into a sum of	

fractions with the same denominator in more than one way and record as an equation. Justify decompositions by using a visual fraction model. • Add and subtract mixed numbers with like denominators by replacing mixed numbers with equivalent fractions, and/or by using properties of operations and the relationship between addition and subtraction. • Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem. b) Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. • Demonstrate a fraction a/b as a multiple of $1/b$. • Generalize a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. • Solve word problems involving multiplication of a fraction by a whole number by using visual fraction models and equations to represent the problem.	
3.3 Illustrate decimal notation for fractions and compare decimal fractions. a) Use decimal notation for fractions with denominators 10 or 100. b) Compare two decimals to hundredths by reasoning about their size. • Recognize comparisons are valid only when two decimals refer to the same whole. • Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, such as using a visual model.	
3.4 Use equivalent fractions as strategy to add and subtract fractions. a) Add and subtract fractions with unlike denominators, including mixed numbers. b) Solve word problems involving addition and subtraction of fractions, including unlike denominators using visual models or equations. • Use benchmark fractions (most common) and number sense (understanding) of fractions to estimate mentally and assess the reasonableness of answers.	
3.5 Apply and extend previous understanding of	

multiplication and division to multiply and divide fractions. a) Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). b) Solve problems using division of whole numbers resulting in fractions or mixed numbers by using visual fraction models or equations. c) Multiply a fraction or whole number by a fraction. d) Interpret multiplication as scaling (resizing) by: • Comparing the size of a product to the size of one factor based on the size of the other factor, without performing the indicated multiplication. • Explaining why multiplying a number by a fraction greater than 1 results in a product greater than the number. • Explaining why multiplying a number by a fraction less than 1 results in a product smaller than the number. • Relating the principle of fraction equivalence $a/b = n \times a)/ n \times b)$ to the effect of multiplying a/b by 1. e) Solve real world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations. f) Divide fractions by whole numbers and whole numbers by fractions. • Interpret division of a fraction by a whole number and compute. • Interpret division of a whole number by a fraction and compute. • Solve real world problems involving division of fractions by whole numbers and whole numbers by fractions by using visual models and equations.	
NRS Level 3 **GE: 4.0 – 5.9**	**NRS Level 4** **GE: 6.0 – 8.9**
MA.ABE.6. **Expressions and Equations**	
3.1 Utilize and extend previous understandings of arithmetic to algebraic expressions. a) Write and evaluate numerical expressions (mathematical phrase using numbers, letters and operations) involving whole-number exponents (power). b) Write, read, and evaluate expressions in which letters stand for numbers. • Write expressions that record operations with	**4.1 Use properties of operations to generate equivalent expressions.** a) Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. b) Describe how rewriting an expression in different forms in a problem can show how the quantities are related.

numbers and with letters standing for numbers. • Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient). • View one or more parts of an expression as a single entity. • Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. c) Perform arithmetic operations in the conventional order when there are no parentheses to specify a particular order (order of operations). d) Apply the properties of operations to generate equivalent expressions. e) Identify when two expressions are equivalent, regardless of which value is substituted into them.	
3.2 Reason and solve one-variable equations and inequalities. a) Solve an equation or inequality as a process of answering a question: • Which values, if any, make the equation or inequality true? • Use substitution to determine an equation or inequality true. b) Use variables to represent numbers and write expressions. • Conclude that a variable can represent an unknown number. c) Solve mathematical and real-world problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. d) Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition. • Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions on number line diagrams.	**4.2 Solve mathematical and real-life problems using numerical and algebraic expressions and equations**. a) Solve multi-step mathematical and real-life problems with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. • Apply properties of operations to calculate with numbers in any form. • Convert between forms as appropriate. • Assess the reasonableness of answers using mental computation and estimation strategies. b) Use variables to represent quantities in a problem, and construct simple equations and inequalities to solve problems. • Solve word problems leading to equations of the form $px + q = r$ and $p(x + q)$ r, where p, q, and r are specific rational numbers. • Solve equations of these forms fluently. • Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. • Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. • Graph the solution set of the inequality and interpret it in the context of the problem.
3.3 Represent and analyze quantitative relationships between dependent and independent variables. a) Use variables to represent two quantities in a real-world problem that change in relationship to one another.	**4.3 Work with integer exponents and radicals** (an expression that has a square root and/or cube root)**.** a) Know and apply the properties of integer **(**a number with no fractional part) exponents to generate equivalent numerical expressions.

b) Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. c) Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	b) Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. • Evaluate square roots of small perfect squares and cube roots of small perfect cubes. • Know that $\sqrt{2}$ is irrational. c) Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. d) Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. • Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities such as using millimeters per year for seafloor spreading. • Interpret scientific notation that has been generated by technology.
	4.4 Build the connections between proportional relationships, lines, and linear equations. a) Graph proportional relationships, interpreting the unit rate as the slope of the graph. b) Compare two different proportional relationships represented in different ways.
	4.5 Analyze and solve linear equations and pairs of simultaneous linear equations. a) Solve linear equations (makes a straight line when graphed) with one variable. • Give examples of linear equations in one variable with one solution, many solutions, or no solutions. • Show these examples by successively transforming the equation into simpler forms, until an equivalent equation of the form x=a, a=a, or a=b results (where a and b are different numbers). • Solve linear equations with rational number coefficients (number used to multiply a variable), including equations that require expanding expressions, using the distributive property, and collecting like terms. b) Analyze and solve pairs of simultaneous linear equations. • Explain that solutions to a system of two linear equations with two variables correspond to points of intersection of their graphs, because

	points of intersection satisfy both equations simultaneously. • Solve systems of two linear equations with two variables algebraically, and estimate solutions by graphing the equations. • Solve simple cases by inspection. • Solve mathematical and real-world problems leading to two linear equations with two variables.
NRS Level 3 **GE: 4.0 – 5.9**	**NRS Level 4** **GE: 6.0 – 8.9**
MA.ABE.7. **The Number System**	
3.1 Compute fluently with multi-digit numbers and find common factors and multiples. a) Fluently divide multi-digit numbers. b) Fluently add, subtract, multiply, and divide multi-digit decimals. c) Find the greatest common factor of two numbers less than or equal to 100. d) Find the least common multiple of two numbers less than or equal to 12. e) Use the distributive property to express a sum of two numbers 1–100 with a common factor as a multiple of the two numbers with no common factor.	**4.1 Apply and extend previous understandings of numbers to the system of rational numbers.** a) Explain positive and negative numbers used to describe quantities having opposite directions or values (temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). • Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. b) Illustrate a rational number as a point on the number line by extending number line diagrams and coordinate axis/axes to represent negative number coordinates. • Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line. • Recognize that the opposite of the opposite of a number is the number itself, such as, (– – 3 = 3), and that 0 is its own opposite. • Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. • Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. • Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. c) Explain ordering and absolute value of rational numbers. • Interpret statements of inequality as statements about the relative position of two numbers on a

	number line diagram. • Write, interpret, and explain statements of order for rational numbers in real-world contexts. • Understand the absolute value of a rational number as its distance from 0 on the number line. • Interpret absolute value for a positive or negative quantity in a real-world situation. • Distinguish comparisons of absolute value from statements about order. d) Solve mathematical and real-world problems by graphing points in all four quadrants of the coordinate plane. • Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
3.2 Utilize and extend previous understandings of multiplication and division to divide fractions by fractions. a) Interpret and compute quotients of fractions. b) Solve word problems involving division of fractions by fractions by using visual models and equations.	**4.2 Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** a) Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers. • Represent addition and subtraction on a horizontal or vertical number line diagram. • Describe situations in which opposite quantities combine to make 0. • Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. • Show that a number and its opposite have a sum of 0 (are additive inverses). • Interpret sums of rational numbers by describing real-world contexts. • Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. • Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. • Apply properties of operations as strategies to add and subtract rational numbers. b) Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

	• Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. • Interpret products of rational numbers by describing real-world contexts. • Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then *–p/q*) = (*–p*)*/q* = *p/(–q).* • Interpret quotients of rational numbers by describing real-world contexts. • Apply properties of operations as strategies to multiply and divide rational numbers. • Convert a rational number to a decimal using long division. • Know that the decimal form of a rational number terminates in 0s or eventually repeats. c) Solve mathematical and real-world problems involving the four operations with rational numbers.
	4.3 Know that there are numbers that are not rational, and approximate them by rational numbers. a. Use rational approximations of irrational numbers to compare the size of irrational numbers. b. Locate them approximately on a number line diagram, and estimate the value of expressions (π 2).
NRS Level 3 **GE: 4.0 – 5.9**	**NRS Level 4** **GE: 6.0 – 8.9**
MA.ABE.8. **Ratios and Proportional Relationships**	
3.1 Develop an understanding of ratio concepts and use ratio reasoning to solve problems. a) Explain the concept of a ratio using ratio language to describe a relationship between two quantities. b) Explain the concept of a unit rate a/b associated with a ratio a:b with b≠0 using rate language in the context of a ratio relationship.	**4.1 Explain ratio concepts and use ratio reasoning to solve problems.** a) Use ratio and rate reasoning to solve mathematical and real-world problems by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. • Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. • Use tables to compare ratios. • Solve unit rate problems including those involving unit pricing and constant speed.

	• Find a percent of a quantity as a rate per 100, such as, 30 % of a quantity is 30/100 time the quantity. • Solve problems involving finding the whole, given a part and the percent. • Use ratio reasoning to convert measurement units. • Manipulate and transform units appropriately when multiplying or dividing quantities.
	4.2 Analyze proportional relationships and use them to solve mathematical and real-world problems. a) Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. b) Recognize and represent proportional relationships between quantities. • Decide whether two quantities are in a proportional relationship by testing for equivalent ratios in a table or graphing on a coordinate plane, and observing whether the graph is a straight line through the origin. • Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. • Represent proportional relationships by equations. • Explain what a point *(x,y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and (1,*r)* where *r* is the unit rate. c) Use proportional relationships to solve multistep ratio and percent problems, such as simple interest, tax, and gratuities.
NRS Level 3 **GE: 4.0 – 5.9**	**NRS Level 4** **GE: 6.0 – 8.9**
MA.ABE.9. **Statistics and Probability**	
3.1 Develop understanding of statistical variability. a) Discuss a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. b) Discuss a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. c) Discuss that a measure of center for a numerical data set summarizes all of its values with a single	**4.1 Summarize and describe distributions.** a) Summarize numerical data sets in relation to their context, such as by: • Reporting the number of observations. • Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. • Giving quantitative measures of center such as median and/or mean.

number, while a measure of variation describes how its values vary with a single number.	• Giving quantitative measures variability such as interquartile range (data divided into quarters) and/or mean absolute deviation (average distance between data value and the mean). • Describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. b) Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
3.2 Summarize and describe distributions. a) Display numerical data in plots on a number line, including: • Dot plots (graph of data using dots). • Histograms (bar graph using ranges of data). • Box plots (graph uses rectangles with lines extending from the top and bottom).	**4.2 Use random sampling to draw inferences about a population.** a) Justify that statistics can be used to gain information about a population by examining a sample of the population. • Generalizations about a population from a sample are valid only if the sample is representative of that population. • Understand that random sampling tends to produce representative samples and support valid inferences. b) Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. c) Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
	4.3 Draw informal comparative inferences about two populations. a) Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities. • Measuring the difference between the centers by expressing it as a multiple of a measure of variability. b) Use measures of center (median and mode) and measures of variability (interquartile range and mean absolute deviation) for numerical data from random samples to draw informal comparative inferences about two populations.
	4.4 Investigate chance processes and develop, use, and evaluate probability models. a) Justify that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. • Larger numbers indicate greater likelihood.

	• A probability near 0 indicates an unlikely event. • A probability around 1/2 indicates an event that is neither unlikely nor likely. • A probability near 1 indicates a likely event. b) Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency. • Predict the approximate relative frequency given the probability. c) Develop a probability model and use it to find probabilities of events. • Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. • Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. • Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. d) Illustrate that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. e) Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. • For an event described in everyday language, such as "rolling double sixes", identify the outcomes in the sample space which compose the event.
	4.5 Investigate patterns of association in data with two variables (bivariate). a. Construct and interpret scatter plots (a graph of plotted points that show the relationship between two sets of data) for bivariate measurement data to investigate patterns of association between two quantities. • Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. b. Know that straight lines are widely used to model relationships between two quantitative variables. • For scatter plots that suggest a linear association, informally fit a straight line, and assess the model fit by judging the closeness of

	the data points to the line. a. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. b. Verify that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. • Use relative frequencies calculated for rows or columns to describe possible association between the two variables.
NRS Level 3 **GE: 4.0 – 5.9**	**NRS Level 4** **GE: 6.0 – 8.9**
MA.ABE.10. **Functions** **Note: Suggested instruction level begins at 7.0 – 8.9**	
Not a focus standard at this level.	**4.1 Define, evaluate, and compare functions.** a) Explain that a function is a rule that assigns to each input exactly one output. • The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required at this level. b) Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line. • Give examples of functions that are not linear.
	4.2 Use functions to model relationships between quantities. a) Construct a function (each input has a single output) to model a linear relationship between two quantities. • Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. b) Describe qualitatively the functional relationship between two quantities by analyzing a graph where the function is increasing or decreasing and linear or nonlinear. • Sketch a graph that exhibits the qualitative

	features of a function that has been described verbally.